Advanced Algorithms — Exercise Set 4

Name: _____

- Submit on Gradescope by class time (1:15pm) on **March 3rd**.
- Feel free to discuss with others, but write up your own work.
- Half points on this exercise set are awarded for completion / effort. Use it to learn!

Graphical Method

Problem 1. The next six questions refer to the following linear program:

$$\begin{aligned} \max \quad & -5x + y \\ \text{s.t.} \quad & -2x + 0.8y \leq 4 && \text{(A)} \\ & 5x + 2y \geq 10 && \text{(B)} \\ & x + 2y \geq 3 && \text{(C)} \\ & x \geq 0 && \text{(D)} \\ & y \geq 0 && \text{(E)} \end{aligned}$$

1. Draw the lines that define the boundary of the feasible region, and shade the feasible region.

2. Determine the optimal solution and its value.

3. Is the feasible region bounded or unbounded? Is the optimal value bounded or unbounded?

4. Determine which constraints are tight at the optimal solution.

5. Suppose we add the constraint $y \leq 3$ to our linear program (A–E). Determine the new optimal solution and its value.

6. Suppose we add the constraint $x \leq 1$ to our linear program (A–E). Determine the new optimal solution and compare it to the one obtained in question 2 above. Did the optimal solution change? Why?

Modeling with Linear Programs

Problem 2. (Trail Mix) You are trying to create the perfect trail mix bag. The ingredients for the trail mix will include Raisins, Grain, Chocolate Chips, Peanuts, and Almonds. These cost, respectively, \$2.50, \$1.50, \$2.00, \$3.50, and \$3.00 per pound. The vitamin, mineral, and protein content of each of the ingredients (in grams per pound) is summarized in the following table, along with the calories per pound of ingredient.

	Raisins	**Grain**	**Chocolate**	**Peanuts**	**Almonds**
Vitamins (g)	30	20	20	40	30
Minerals (g)	6	5	6	10	4
Protein (g)	6	4	3	15	2
Calories	400	180	600	400	450

You would like to identify the least costly mix of these ingredients that provides at least 44 grams of vitamins, 15 grams of minerals, 12 grams of protein, and 550 calories for a package that must weigh exactly 2 pounds. Formulate a linear programming model for this problem, including variable definitions, an objective function, and constraints.

Modeling with Integer Programs

Problem 3. (Knapsack) Suppose we are given a set of n items, each with a value and a weight. The value of item $i \in [n]$ is v_i and its weight is denoted w_i. We also have a backpack which can carry items up to a total weight at most W. The goal is to select a subset of items of total weight at most W such that the value of selected items is maximized. Formulate this optimization problem as an Integer Program.

Problem 4. (TSP) Given a undirected complete graph $G = (V, E)$, with non-negative costs c_e on edges, the Traveling Salesperson Problem asks for a Hamiltonian cycle of minimum total cost. A cycle is Hamiltonian if it visit every vertex exactly once.

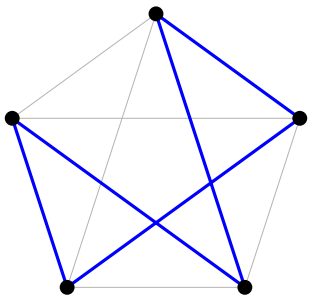

Figure 1: Complete graph on 5 nodes with a Hamiltonian cycle shown in blue.

Try formulating the Traveling Salesperson Problem as an integer program.

Hint: TSP can be phrased as finding a subset of edges $F \subseteq E$ of minimum total cost such that (V, F) is connected and all vertices have degree 2.

Problem 5. Fill out the mid-semester feedback form! This is posted on the course webpage.